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FORECASTING INTEREST RATE VOLATILITY IN NIGERIA IN THE ARCH-GARCH FAMILY MODELS

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Abstract: Modeling the volatility of interest rates is essential for many areas in finance. However, it is well known that interest rate series exhibit non-normal characteristics that may not be captured with the standard GARCH model with a normal error distribution. But which GARCH model and error distribution to use is still open to question, especially where the model that best fits the in-sample data may not give the most effective out-of-sample volatility forecasting ability, which we use as the criterion for the selection of the most effective model from among the alternatives. In this work, the GARCH family models were employed in modeling interest rate volatility in Nigeria. A time series of data spanning January 2000 to December 2018 (in-sample data) was used to fit the models and out-of-sample data running from January to December 2019 to determine the best conditional volatility forecast. Twenty-four symmetric and eighteen asymmetric models were estimated and compared using three distribution errors; the normal, student's t, and the generalized error distributions; while four error loss functions, namely, RMSE, MAE, MAPE, and TIC, were adopted to determine the best fit and conditional volatility forecast. The result shows that the symmetric GED-GARCH (1,1) model was considered the overall best fit in both the symmetric and asymmetric models. The best-fitted GED-GARCH (1,1) model exhibited volatility persistence. The in-sample and out-of-sample volatility forecast of the GED-GARCH (1,1) model reveals that unconditional mean and variance will be achieved in the third month of 2019. Some transmission spillover effects running from the exchange and inflation rates to interest rates were also detected.

Keywords: Interest rate, Volatility, GARCH-Type Models, Best fit, Persistence, forecast

INTRODUCTION

The fluctuation of a variable over a period of time is an indication of the volatility of that variable, and the deviation from an expected value is often used to describe volatility. Financial volatility is defined as the measure of the variation in the price of a financial instrument over time (Ezzati, 2013). One of the basic roles of the Central Bank of Nigeria in controlling financial institutions is the setting of interest rates. The economic well-being of any nation is, to a large extent, determined by the interest rate fixed by its apex bank. We know that the interest rate and the aggregate supply of money in circulation are the two main instruments of monetary policy, which can either be achieved by controlling the growth of money supply or expanding the supply of money in circulation, which in turn leads to excess demand, thereby causing the interest rate

to decline. Although the 1980 economic reforms saw some significant levels of development, particularly in the financial system, there are still many unresolved economic problems, particularly interest rates, which have had devastating effects on the cost of borrowing and investment in Nigeria and which have been the bane of dissatisfied foreign investment. The interest rate policy in Nigeria, for example, has changed within the time frame of regulated and deregulated regimes. According to (Okwori et al., 2014), Nigeria has pursued a two-interest rate regime starting from the 1960s to the mid-1980s with the administration of low interest rates, which was intended to encourage investment. However, the advent of the Structural Adjustment Programme (SAP) in the third quarter of 1986 ushered in an era when fixed and low-interest rates were gradually replaced by a dynamic interest rate regime, where rates were more influenced by market forces. (Chirwa E.W. and M. Mlachila, 2004) argued that the major economic indicator used to boost investment is interest rates, which have been found to be higher in Africa, Latin America, and the Caribbean countries than in the Organization of European Countries. The behavior of interest rates, to a large extent, determines the investment activities and hence economic growth of a country. According to (Jhingan, 2003), if interest rates are high, investment is at a low level; when interest rates fall, the investment will rise. The high-interest rate in Nigeria might be owing to high inflation that remained at double digits and other macroeconomic factors like the instability in the Nigeria currency, even the increased sub-national government spending and government high expenditure. On the basis of the foregoing, it becomes necessary to investigate the dynamic nature of interest rates in Nigeria and whether or not there are external forces influencing such volatility, so that investors and government agencies can, on the basis of the outcome of this research, be properly informed in making appropriate investment and policy decisions.

The economic status of any nation is, to a large extent, determined by the level of investment made by the private sector, foreign investments, and the national government. Following the assertion that interest rates (bank lending rates) are highly volatile in Nigeria, both local and foreign investors have become skeptical about whether to borrow and invest or not and when. It was also argued that there may be other independent variables whose variances may be contributing to the conditional volatility of interest rates in Nigeria over the years. It, therefore, becomes imperative to study this dynamism or volatility of interest rates in Nigeria so that these investors can rightly be advised on what the future holds. To accomplish this, it is necessary to look for a statistical package or GARCH-family model that will provide the best fit for the best forecast for such advice.

Statement of the Problem

The economic status of any nation is, to a large extent, determined by the level of investment made by the private sector, foreign investments, and investment by the national or federal government. Following the assertion that interest rates (bank lending rates) are highly volatile in Nigeria, both local and foreign investors have become skeptical about whether to borrow and invest or not and when. To advise these investors, a statistical package or GARCH-family model that provides the best fit for such advice is required. If interest rates are indeed volatile, are we certain that the variances of other related economic variables are not equally affecting the conditional variance of interest rates in Nigeria, i.e. the spillover effect? It is therefore critical to investigate the dynamism or volatility of interest rates in Nigeria so that these investors can be properly advised on what the future holds. However, while modeling the volatility of interest rates is essential for many areas in finance, it is well known that interest rate series (like many other variables) exhibit non-normal characteristics that may not be captured with the standard GARCH model with a normal error distribution. But which GARCH model and error distribution to use is still open to question, especially where the model that best fits the in-sample data may not give the most effective out-of-sample volatility forecasting ability, which we use as the criterion for the selection of the most effective model from among the alternatives.

1. **Theoretical Review of Related Literature**

The first breakthrough in volatility modeling was (Engle, 1982), where it was shown that conditional heteroscedasticity can be modeled using the Autoregressive Conditional Heteroscedasticity (ARCH) model. The ARCH model relates the conditional variance of the disturbance term to the linear combination of the squared disturbance in the recent past Determining optimal lag length is cumbersome, often times engendering parameterization. However, (Bollerslev, 1986) and Taylor (1986) independently proposed the extension of the ARCH model with an Autoregressive Moving Average (ARMA) formulation, with a view to achieving parsimony. The model is called the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model, which models conditional variance as a function of its lagged values as well as squared lagged values of the disturbance term. Although the GARCH model has proven useful in capturing symmetric effects of volatility, it is bedeviled with some limitations, such as the violation of non-negativity constraints imposed on the parameters to be estimated.

To overcome these constraints, some extensions of the original GARCH model were proposed. This includes asymmetric GARCH family models such as Threshold GARCH (TGARCH) proposed by (Zakoian, 1994).

Exponential GARCH (EGARCH) was proposed by Nelson (1991) and Power GARCH (PGARCH) was proposed by Ding et al (1993). The idea of the proponents of these models is based on the understanding that good news (positive shocks) and bad news (negative shocks) of the same magnitude have different effects on conditional variance. The EGARCH model which captures asymmetric properties between interest rate and conditional volatility was proposed to address major deficiencies of GARCH model. They are (i) Parameter restrictions that ensure conditional variance positivity; (ii) non-sensitivity to asymmetric response to shocks and (iii) difficulty in measuring persistence in a strongly stationary series. The log of the conditional variance in the EGARCH model signifies that the leverage effect is exponential and not quadratic.

2. Empirical Review of Related Literature

Several empirical works have been done since the seminar paper of (Engle, 1982) on volatility modeling, especially in finance, even though a number of theoretical issues still remain unresolved (Franses & McAleer, 2002). (Olweng, 2011) studied the best fit of short-term interest rate volatility in Kenya. The ARCH(q) and GARCH (p,q) models were considered and the result revealed that the GARCH (p,q) model is better for explaining the conditional volatility of short-term interest rates in Kenya. The result further indicated the presence of volatility clustering that exists as a link between the level of short-term interest rate and volatility of interest rate in that country. He recommended that the study be extended to asymmetric GARCH models.

(Bayracı & Ünal, 2014) in their work to determine the best fit applied the discrete-time

GARCH (1,1) and continuous-time COGARCH (1,1) models to analyze the interest rate dynamics in Turkish market. The result shows that the COGARCH (1,1) model provided the best and excellent result in modeling interest rate series, as they capture the characteristics of the volatility process and yielded a better conditional volatility estimate than the discrete-time counterpart GARCH (1,1) model.

(Okoro et al., 2017) applied two asymmetric models, EGARCH (1,1) and GJR-GARCH(1,1) models in the forecasting of USDNGN Exchange rate in Nigeria, under error distributions such as the normal, skewed normal, student's t-distribution, skewed student's t-distribution, Generalized error distribution and skewed Generalized error distribution. The result obtained indicates that all the models performed fairly well in capturing the volatility fluctuation of Nigeria Exchange rate returns with slight advantage of GED-EGARCH (1,1) and GJR-

GARCH (1,1) for the in-sample fit. The two models have the lowest AIC and the highest log likelihood values. For out-of-sample forecasting, the EGARCH (1,1) analyzed with Generalize Error Distribution have the minimum MSE and MAE respectively. The empirical results of the study however revealed evidence of leverage effects in USDNGN Exchange rate return within the period under study. (Omari-Sasu et al., 2015), studied the volatility of stock market in Ghana and employed the GARCH family model to determine the best model that will best explain the stock market in that country. The result shows that the GARCH (1,1) model was the best fit among others in the analysis of three equities examined. The work further revealed that though there is a presence of volatility, but not persistence in the three stock markets examined. (Kosapattarapim et al., 2012), in evaluating the volatility forecasting performance of best-fitting GARCH models in emerging Asian stock markets concluded that out of six different types of error distributions employed in the analysis, the GARCH (p,q) model with non-normal error distribution tend to provide out-of-sample forecast performance than a GARCH (p,q) model analyzes with normal error distribution.

(Tobia, 2011) inferred that there is a relationship between interest rate and interest rate volatility in Kenya. The work further noted that GARCH (1,1) model is ideal for modeling interest rate volatility in Kenya compared to other GARCH family models studied.

(Ahmed & Suliman, 2011), examined modelling stock market volatility using GARCH models evidence from Sudan. The symmetric and asymmetric behavior of the stock was analyzed and the result revealed that the conditional variance process was highly persistent and as such provided evidence of risk premium for the KSE index stock series which showed that the asymmetric model provided a better fit than the symmetric model, which confirmed the presence of leverage effect.

(Maqsood et al., 2017) in their work employed the GARCH model to analyze the stock market volatility of Nairobi Securities Exchange (NSE). According to the report, the GARCH process captured the symmetric and asymmetric properties of the models and in agreement with the work done by inferred that the volatility process is highly persistent, showing evidence of risk premium for the NSE index return series. The report further revealed that the symmetric model provided a better fit than the asymmetric model.

(Ahmed & Suliman, 2011), in their work applied the GARCH models to forecast the stock market volatility. Contrary to the works of (Maqsood et al., 2017) and, inferred that on the basis of out-of-sample forecasts and a majority of evaluation measures, the asymmetric GARCH model performed better in forecasting conditional variance of the BSE-SENSEX returns than the symmetric GARCH model, confirming the presence of leverage effect. Dedi and Yavas, (2016) used the Augmented GARCH model to detect the spillover effect between markets. Similarly, (Edwards, 1998) and Zouch, Abbes, and Boujelbene (2011) used the Augmented GARCH model and detected the presence of capital transmission/spillover effect Mexico to

Chile for the Mexico bonds during 1994 crisis.

MATERIALS AND METHODS

In financial time series, modelling real data needs proper attention, and suitable model selection is also required to better understand the structure of the statistical data which ultimately helps in better forecasting. This is because these selected models are later used for policy-making whether in finance or economics. The reason for this care is the non-linear dynamics present in such data. For financial data, it is sometimes obvious to find volatility clusters in a given set of data. According to (Mandelbrot, 1963) volatility clustering refers to the observation where large changes tend to be followed by large changes of either sign, and small changes tend to be followed by small changes. In other words, volatility depends more on recent past values than distant past values.

1. **Model Specification.**

Financial markets react nervously to political disorder, economic crises, wars or natural disasters. Similarly, following the rate of inflation, volume of money supply, exchange rate and other regulatory instruments employed by Central Bank of Nigeria (CBN) to ensure sustained economic growth, the interest rate no doubt fluctuates at low and high rates at interval of times. To model time series volatility, the symmetric ARCH-GARCH family models shall be employed in the analysis of this work.

2. **Autoregressive Conditional Heteroscedasticity (ARCH) Model**

The ARCH method provides a way to model a change in variance in a time series that is time dependent, such as increasing or decreasing volatility. Autoregressive therefore describes a feedback mechanism that incorporates past observations into the present, that is, the series depends on its past values. In other words, it implies that (unequal variance) observed in the series over different time periods may be auto-correlated. Conditional implies that variance is based or depends on past errors (shocks). ARCH therefore simply conveys that series in question which has a time-varying variance that depends on the lagged effects (autocorrelation). The ARCH model originally proposed by (Engle, 1982) is given by;

$$\sigma^2_t = \varepsilon_t \sigma_t \quad (3.1)$$

where $\varepsilon_t \sim N(0, 1)$, $t = 1, 2, \ldots n$ and equation (3.1) is called the mean equation while the ARCH variance equation is given by;

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-i}^2 \quad (3.2)$$

$where$ $\sigma_t|\Psi_t \sim N(0, \sigma)$ and σ_t is the dependent variable, α_0 is the constant term, ε_t is the disturbance term, Ψ_t is the information set available at time t, q is the lag length of ARCH model and $\boldsymbol{\alpha}^{'s}$ vectors of unknown parameters in the variance equation.

3. **Test for ARCH-GARCH Effect.**

Though it is assumed that financial and economic variables change rapidly from time to time in an apparently unpredictable manner, it therefore becomes necessary to determine periods when large changes are followed by further large changes and periods when small changes are followed by further small changes, popularly known as volatility clustering.

An estimation test to determine whether a particular variable or series is volatile (has ARCH effect) or not becomes necessary. To achieve this, a methodology to test for lag length of ARCH errors using the Lagrange Multiplier (LM) test was proposed by (Engle, 1982).

The Lagrange Multiplier (LM) test statistic is defined as;

$$LM = T^{l}R^{2} \sim \chi^{2}_{(q)} \quad (3.3)$$

Where T^{l} is the number of equations in the model which fits the residual versus the lags, that is, T^{l} = T-q, where q is the lag length of the ARCH model. The null hypothesis is rejected in favour of the alternative if the p-value is less than one or five percent level of significance.

4. Generalized Autoregressive Conditional Heteroscedasticity (GARCH) Model

The generalized autoregressive conditional heteroscedasticity (GARCH) model, is an extension of the ARCH model that incorporates a moving average component together with the autoregressive component. The introduction of the moving average component is to allow the model to both model the conditional change in the variance over time as well as changes in the time-dependent variance.

(Bollerslev, 1986) extended Engle's original work by developing a technique that allows conditional variance to be an ARIMA process. If we allow the error process to be such that;

$$\sigma_t^{2} = \varepsilon_t\sigma_t \quad (3.4)$$

where $\varepsilon_t \sim N(0,1)$, $t = 1, 2, \ldots n$ and

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \, \sigma_{t-1}^2 \quad (3.5)$$

is defined as the generalized ARCH (p, q) model. σ_t^{2} is the conditional variance, α_0 is the constant term, α_i are the coefficients of the squared error of the ARCH component while β_i are the coefficients of the conditional variance of the GARCH term. $\varepsilon_t^{2}{}_{-1}$ measures the shock on volatility. The conditional term compared with the ARCH(q) model is the forecasted variance from the previous period given by σ_{t-1}^2.In other words, the GARCH model is a model that attempts to explain the conditional volatility using the past lagged squared errors (ε_{t-1}^2) and the past conditional variance(σ_{t-1}^2). However, a typical GARCH (1,1) model Equation is given by;

$$\sigma_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2 \quad (3.6)$$

5. Non-negativity Constraints and Stationarity in GARCH (1,1) Model

Brooks (2008) inferred that the values of a conditional variance must always be strictly positive; a negative variance at any point in time would be meaningless. The variable on the RHS of the conditional variance equation are all squares of lagged errors, and so by definition will not be negative. In order to ensure that these always result in positive conditional variance, all of the coefficients in the conditional variance are required to be non-negative. This non-negativity therefore implies that $\alpha_0 > 0$, $\alpha_1 \geq 0$, $\beta_1 \geq 0$. Going by the above, the stationarity condition of a standard GARCH model states that $\alpha_0 > 0$, $\alpha_1 \geq 0$, $\beta_1 \geq 0$, and, $\alpha_1 + \beta_1 < 1$. The achievement of these conditions implies the model is well-defined. The unconditional variance under GARCH model specification indicated that the conditional variance of ε_i is constant and given by;

$$Var(\varepsilon_t) = \frac{\alpha_0}{1-\alpha_1-\beta}$$

(3.7)

6. Asymmetry Volatility Models

However, neither the ARCH (q) nor the GARCH (p, q) is able to incorporate the asymmetry volatility. To adjust for this condition, several models have been developed using the GARCH model as their foundation. Other GARCH family models employed to determine the Asymmetric properties of the model and best fit are as follows:

$$\gamma \frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}} \left[\frac{|\varepsilon_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right]$$

(A) Exponential Generalized Autoregressive Conditional Heteroscedasticity (EGARCH)

The EGARCH model was the first model to incorporate asymmetry volatility. Empirical studies have shown that the EGARCH provides a more accurate result compared to the conventional symmetric ARCH and GARCH models, (Alberg. shalit, Yosef, 2008).

(Adeleye, 2019), inferred that news, incidents, merger of companies, acquisition of companies, wars, terrorist attacks, launch of new discoveries, secession or independence etc, have strong and powerful influence on the decision making of financial investors, hence, have asymmetric impact on financial investors across the globe.

Hence, the impact of good and bad news on financial market is asymmetric (not the same). In asymmetric models, positive shocks do not have exactly the same magnitude with negative shock and vice versa.

The EGARCH variance equation with normal distribution is hereby stated (Brooks, 2014).

$$\log(\sigma^2) = \omega + \beta \log(\sigma_{t-1}^2) + \gamma \frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha \left[\frac{|\varepsilon_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right]$$

(3.8)

Where ω is the intercept for the vaeiance, β is the coefficient for the logged GARCH term, $\log(\sigma_{t-1}^2)$ is logged GARCH term, γ is the scale of the asymmetric volatility, is the last period shock which is standardized, and parameter that takes into account the absolute value of the last period's volatility shock. It replaces the regular ARCH term. The model captures the asymmetry volatility through the variable gamma (γ). The sign of the gamma determines the size of the asymmetric volatility, and if the asymmetric volatility is positive or negative (Brooks, 2014).

If γ = 0, implies is symmetric or no asymmetric volatility. If γ < 0, implies negative shock will increase the volatility more than positive shocks. If γ > 0, implies that positive shock increases the volatility more than negative shocks. According to previous studies in the subject, the coefficient γ is often negative, this implies that negative shocks have more impact on volatility than positive shocks (good news). Given that the model uses the log of the variance (σ^2), this means that even if the parameters are negative, the variance will still be positive. Therefore, the model is not subject to the non-negative constraints. (B) Threshold GARCH Model (TGARCH)

The GJR-GARCH also called Treshold

GARCH (TGARCH) model was developed by Glosten, Jagannathan & Runkle (1993). The advantage of the model is that the variance is directly modelled and does not use the natural logarithm like the EGARCH model. The main target of the TGARCH model is to capture asymmetries in terms of negative and positive shocks. To do that is simply to add into the variance equation of the GARCH model a multiplicative dummy variable $\gamma\varepsilon_{t-i}I_{t-i}$ to check whether there is statistically significant difference when the shocks are negative. Hence the conditional variance for a TGARCH or GJR-GARCH is given by:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i\, \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j\, \sigma_{t-1}^2 + \sum_{i=1}^{m} \gamma \quad (3.9)$$

However, the form of GJR-GARCH (1, 1) is given by:

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-i}^2 + \beta\sigma_t^2 + \gamma\varepsilon_{t-i}I_{t-i} \quad (3.10)$$

In equation (3.9) above, σ_t^2 is the conditional forecasted variance, ω is the intercept for the variance, while $\alpha\varepsilon_{t-i}^2$ is the variance that depends on previous lag error term. β is the coefficient of previous period forecasted variance and σ_{t-1}^2 is the previous period forecasted variance. Moreso, γ is the scale (coefficient) of the asymmetric volatility. I_{t-i} is a dummy variable that is only activated if the previous shock is negative (ε_{t-i}< 0), allowing the GJR-GARCH to take the leverage effect into consideration. Glosten et al (1993).

$$I_{t-i} = \begin{cases} 1 \text{ if } \varepsilon_{t-i} < 0 \text{ (Negative shock)} \\ 0 \text{ otherwise (Positive shock)} \end{cases}$$

From equation (3.10), good news, (positive shock) and bad news, that is, negative shock (ε_{t-i}< 0) have different impacts on the conditional variance. A positive shock is captured by the coefficient α, ie have an α -effect on the conditional variance σ_t^2, while negative shock (Bad news) has an $(\alpha + \gamma)$ effect on the conditional variance σ_t^2, (volatility). If γ = 0, the GJR-GARCH model becomes a linear symmetric GARCH model but if γ $\neq 0$, then that suggests an asymmetric (leverage) effect. However, if al so: $\gamma > 0$, means negative shocks will increase volatility more than positive shock. If γ < 0, positive shock will increase volatility more than negative shocks.

(C) The Power GARCH (PGARCH) Model

Ding et al (1993), expressed conditional variance using PGARCH (p,d,q) as;

$$\sigma_t^d = \beta_0 + \sum_{i=1}^{p} \alpha_i(|\varepsilon_{t-1}| + \gamma\varepsilon_{t-1})^d + \sum_{j=1}^{q} \beta_j\sigma_{t-j}^d \quad (3.11)$$

Here, d > 0 and $\gamma_i < 1$ establishes the existence of leverage effects. If d is set at 2, the PGARCH (p , q) replicates a GARCH(p , q) with a leverage effect. If d is set at 1, the standard deviation is modelled. The first order of the above PGARCH equation is PGARCH (1, d, 1) expressed as:

$$\sigma_t^d = \beta_0 + \alpha_1(|\varepsilon_{t-1}| + \gamma\varepsilon_{t-1})^d + \beta_1\sigma_{t-1}^d \quad (3.12)$$

The failure to accept the null hypothesis that $\gamma_1 \neq 0$ shows the presence of leverage effect. The impact of news on volatility in PGARCH is similar to that of TGARCH when d is 1

6. Volatility Transmission (Spillover Effect)

The transmission of shocks from one

market or variable to another was well documented by (Ewing, 2002). Co-movement across volatilities (co-volatility) due to common information that simultaneously affect expectations and information spillover caused by cross-market hedging are some of the reasons for volatility transmissions. In addition to endogenous events or variables, exogenous variables, deterministic events (macroeconomic announcements) may all have influence on the volatility process. To determine volatility transmission (Spillover) between variables (markets), we use the Augmented GARCH model as developed by (Duan, 1997). The model is defined as follows:

$$\sigma_t^2 = \alpha_0 + \beta\sigma_{t-1}^2 + \alpha\varepsilon_{t-1}^2 + \theta X_t \quad (3.13)$$

Where X_t is the residual squared error of ARMA model and θ the term that measures the magnitude of volatility spillover (transmission) across the variables (markets). Two variables, namely, exchange and inflation rates were introduced to determine whether they have any spillover effect on the conditional volatility of interest rate in Nigeria within the period under review.

7. Model Selection Criteria

Finding optimal of a model that will fit a particular data set has always been favourite for researchers. Reinhard and Lunde (2001), inferred that there is not a unique criterion for selecting the best model, rather it depends on preferences, example, expressed in terms of a utility function or loss function. The standard model selection criteria of (Akaike et al., 1973) and Schwartz are often applied. Bieren, H, J (2006) recommended the following modification, if the model includes ARCH type errors.

$$AIC = -2\log(\sigma^2) + 2k - 1 - \log(2\pi) \quad (3.14)$$

Shibata (1976) showed through empirical evidence that AIC has the tendency to choose models which are over parameterized. Various modifications have been produced to overcome this lack of consistency. (Schwarz, 1978) developed a consistent criterion for models defined in terms of their posterior probability (Bayesian approach) which is given by;

$$SIC = -2log(\sigma^2) + klog(n) \quad (3.15)$$

Where $(\widehat{\sigma^2})$ is the estimated model error variance, k is the number of free parameters in the model, n is the number of observations. In ARCH context, this form will look like;

$$SIC = -2log(\widehat{\sigma^2}) + klog(n) - 1 - \log(2\pi) \quad (3.16)$$

8. Loss Functions-Measure of Forecast Performance

Although in literature, several methods of measuring the performance conditional variance,

(Liu, 2009) inferred that the Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Mean Absolute Percent Error (MAPE) and Theil Inequality Coefficient (TIC) are most appropriate in determining the best forecast performance. (Clements, 2005), on the predictive ability of volatility models proposes that out-of-sample forecasting ability remains the criterion for selecting the best predictive model, hence shall be adopted in this study. If $\sigma_t{}^2$ and $\widehat{\sigma}_t^2$ represents the actual and forecasted volatility of interest rate at time t, then;

$$RMSE = \sqrt{\frac{1}{T}\sum_{i=1}^{T}(\sigma_t^2 - \sigma_t^2)}$$

(3.17)

$$MAE = \frac{1}{T}\sum_{I=1}^{T}|\hat{\sigma}_t^2 - \sigma_t^2|$$

(3.18)

$$MAPE = \frac{1}{T}\frac{\sum_{i=1}^{T}(\widehat{\sigma}_t^2 - \sigma_t^2)}{Z_t}$$

(3.19)

$$TIC = \frac{\sqrt{\frac{1}{T}\sum_{i=1}^{T}(\widehat{\sigma}_t^2 - \sigma_t^2)^2}}{\sqrt{\frac{1}{T}\sum_{i=1}^{T}(\widehat{\sigma}_t^2)^2} - \sqrt{\frac{1}{T}\sum_{i=1}^{T}(\sigma_t^2)^2}}$$

(3.20)

The volatility model with the least RMSE, MAE, MAPE and TIC statistic is the best forecasting model.

9. Distribution of Errors $\boldsymbol{\varepsilon}_t$

As far as error distribution is concerned, GARCH model theory suggests three assumptions about the distribution of residuals. These three assumptions may follow normal law, a student law or a generalized Error distribution (GED). In this work three different distributions namely; the normal (Gaussian), the student's t-Distribution and the Generalize Error distribution were employed to determine the best fit and forecast of the conditional variance. The Normal distribution:

$Z2$

$$f(z) = \frac{1}{\sqrt{2\pi}}e^{-\frac{}{2}}, \qquad -\infty < Z < \infty$$

(3.21)

$$f(Z) = \frac{\Gamma(\frac{v+1}{2})}{\sqrt{v\pi\Gamma(\frac{v}{2}}}\left(1 + \frac{Z^2}{v}\right)^{-\frac{(v}{2}}$$

Student t-distribution:

+1)

$-\infty < Z < \infty$ (3.22)

Generalized Error Distribution:

$$\sigma^{-1}ve^{(-0>5)}\left|\frac{\left(\frac{Z-\mu}{\sigma}\right)}{\lambda}\right|$$

$$f(Z,\mu,\sigma,v) = \frac{\bar{v}}{\lambda^{\ (1+\frac{1}{v})}\ (^{1})} \quad 2 \quad \Gamma\,\bar{v}$$

(3.23)

V > 0 is the degree of freedom or the tail-tickness.

10. Mean Reversion

(John et al., 2019), stated that mean reversion means that current information has no influence on the long run forecast of the volatility. Persistence dynamics in volatility is generally captured in the GARCH coefficients of a stationary GARCH-type model. In stationary GARCH-type models, the volatility mean reverts to its long-run level, at a rate given by sum of ARCH and GARCH coefficients, which is usually close to one (1) for financial time series. The average number of time periods for the volatility to revert to its long run level is measured by the half-life of the volatility shock. The mean reverting form of the GARCH (1,1) model is given by;

$$\varepsilon_t{}^2 - \bar{\sigma}^2 = (\alpha + \beta)(\varepsilon_t{}^2{}_{-1} - \bar{\sigma}^2) + r_1 + \beta_1 + r_{1-1} \qquad (3.24)$$

Where $\bar{\sigma}^2 = \frac{\alpha_0}{1-\alpha-\beta}$, is the unconditional long-run level of volatility and $r_1 = (\varepsilon_t^2 - \bar{\sigma}^2)$. The magnitude of the mean reverting rate $\alpha + \beta$ (speed of adjustment) controls the speed of the mean reversion.

RESULTS AND DISCUSSION

In financial time series, modelling real data needs proper attention, and suitable model selection is also required to better understand the structure of the statistical data which ultimately helps in better forecasting. This is because these selected models are later used for policy making whether in finance or economics. The reason for this care is the non-linear dynamics present in such data. For financial data, it is sometimes obvious to find volatility clusters in a given set of data. According to (Mandelbrot, 1963) volatility clustering refers to the observation where large changes tend to be followed by large changes of either sign, and small changes tend to be followed by small changes. In other words volatility depends more on recent past values than distant past values.

1. **Model Specification.**

Financial markets react nervously to political disorder, economic crises, wars, or natural disasters. Similarly, following the rate of inflation, volume of money supply, exchange rate and other regulatory instruments employed by the Central Bank of Nigeria (CBN) to ensure sustained economic growth, the interest rate no doubt fluctuates at low and high rates at intervals of times. To model time series volatility, the symmetric ARCH-GARCH family models shall be employed in the analysis of this work.

2. **Autoregressive Conditional Heteroscedasticity (ARCH) Model**

The ARCH method provides a way to model a change in variance in a time series that is time-dependent, such as increasing or decreasing volatility. Autoregressive, therefore, describes a feedback mechanism that incorporates past observations into the present, that is, the series depends on its past values. In other words, it implies that (unequal variance) observed in the series over different time periods may be auto-correlated. Conditional implies that variance is based or depends on the past errors (shocks). ARCH therefore simply conveys that series in question which has time-varying variance that depends on the lagged effects (autocorrelation). The ARCH model originally proposed by (Engle, 1982) is given by;

$$\sigma^2{}_t = \varepsilon_t \sigma_t \quad (3.1)$$

where $\varepsilon_t \sim N(0, 1)$, $t = 1, 2, \dots n$

and equation (3.1) is called the mean equation while the ARCH variance equation is given by;

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-i}^2 \quad (3.2)$$

where $\sigma_t | \Psi_t \sim N(0, \sigma)$ and σ_t is the dependent variable, α_0 is the constant term, ε_t is the disturbance term, Ψ_t is the information set available at time t, q is the lag length of ARCH model and $\boldsymbol{\alpha}$'s vectors of unknown parameters in the variance equation.

3. **Test for ARCH-GARCH Effect.**

Though it is assumed that financial and economic variables change rapidly from time to time in an apparently unpredictable manner, it therefore becomes necessary to determine periods when large changes are followed by further large changes and periods when small changes are followed by further small changes, popularly known as volatility clustering.

An estimation test to determine whether a particular variable or series is volatile (has ARCH effect) or not becomes necessary. To achieve this, a methodology to test for lag length of ARCH errors using the Lagrange Multiplier (LM) test was proposed by (Engle, 1982).

The Lagrange Multiplier (LM) test statistic is defined as;

$$LM = T^{l}R^{2} \sim \chi^{2}_{(q)} \quad (3.3)$$

Where T^{l} is the number of equations in the model which fits the residual versus the lags, that is, T^{l} = T-q, where q is the lag length of the ARCH model. The null hypothesis is rejected in favour of the alternative if the p-value is less than one or five percent level of significance.

4. **Generalized Autoregressive Conditional Heteroscedasticity (GARCH) Model**

Generalized autoregressive conditional heteroscedasticity (GARCH) model, is an extension of the ARCH model that incorporates a moving average component together with the autoregressive component. The introduction of the moving average component is to allow the model to both model the conditional change in the variance over time as well as changes in the time dependent variance.

(Bollerslev, 1986) extended Engle's original work by developing a technique that allows the conditional variance to be an ARIMA process. If we allow the error process to be such that;

$$\sigma_t^2 = \varepsilon_t \sigma_t \quad (3.4)$$

where $\varepsilon_t \sim N(0,1)$, $t = 1, 2, \dots n$ and

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \, \sigma_{t-1}^2 \quad (3.5)$$

is defined as the generalized ARCH (p, q) model. σ_t^2 is the conditional variance, α_0 is the constant term, α_i are the coefficients of the squared error of the ARCH component while β_i are the coefficients of the conditional variance of the GARCH term. $\varepsilon_t^2{}_{-1}$ measures the shock on volatility. The conditional term compared with the ARCH(q) model is the forecasted variance from the previous period given by σ_{t-1}^2.In other words, the GARCH model is a model that attempts to explain the conditional volatility using the past lagged squared errors (ε_{t-1}^2) and the past conditional variance(σ_{t-1}^2). However, a typical GARCH (1,1) model Equation is given by;

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (3.6)$$

5. **Non-negativity Constraints and Stationarity in GARCH (1,1) Model**

Brooks (2008) inferred that the values of a conditional variance must always be strictly positive; a negative variance at any point in time would be meaningless. The variable on the RHS of the conditional variance equation are all squares of lagged errors, and so by definition will not be negative. In order to ensure that these always result in positive conditional variance, all of the coefficients in the conditional variance are required to be non-negative. This non-negativity therefore implies that $\alpha_0 > 0$, $\alpha_1 \geq 0$, $\beta_1 \geq 0$. Going by the above, the stationarity condition of a standard GARCH model states that $\alpha_0 > 0$, $\alpha_1 \geq 0$, $\beta_1 \geq 0$ and, $\alpha_1 + \beta_1 < 1$. The achievement of these conditions implies the model is well defined. The unconditional variance under

GARCH model specification indicated that the conditional variance of ε_i is constant and given by;

$$Var(\varepsilon_t) = \frac{\alpha_0}{1-\alpha_1-\beta}$$

(3.7)

6. Asymmetry Volatility Models

However, neither the ARCH (q) nor the GARCH (p, q) is able to incorporate the asymmetry volatility. To adjust for this condition, several models have been developed using the GARCH model as their foundation. Other GARCH family models employed to determine the Asymmetric properties of the model and best fit are as follows:

(A) Exponential Generalized Autoregressive Conditional Heteroscedasticity (EGARCH)

The EGARCH model was the first model to incorporate asymmetry volatility. Empirical studies have shown that the EGARCH provides a more accurate result compared to the conventional symmetric ARCH and GARCH models, (Alberg shalit, Yosef, 2008).

(Adeleye, 2019), inferred that news, incidents, merger of companies, acquisitions of companies, wars, terrorist attacks, launch of new discoveries, secession or independence etc, have strong and powerful influence on the decision making of financial investors, hence, have asymmetric impact on financial investors across the globe.

Hence, the impact of good and bad news on financial market is asymmetric (not the same). In asymmetric models, positive shocks do not have exactly the same magnitude with negative shock and vice versa.

The EGARCH variance equation with normal distribution is hereby stated (Brooks, 2014).

$$\log(\sigma^2) = \omega + \beta \log(\sigma_{t-1}^2) + \gamma \frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha \left[\frac{|\varepsilon_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right]$$

$$\left[\frac{|\varepsilon_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right] \gamma \frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}}$$

(3.8)

Where ω is the intercept for the vaeiance, β is the coefficient for the logged GARCH term, $\log(\sigma_{t-1}^2)$ is logged GARCH term, γ is the scale of the asymmetric volatility, is the last period shock which is standardized, and parameter that takes into account the absolute value of the last period's volatility shock. It replaces the regular ARCH term. The model captures the asymmetry volatility through the variable gamma (γ). The sign of the gamma determines the size of the asymmetric volatility, and if the asymmetric volatility is positive or negative (Brooks, 2014).

If γ = 0, implies is symmetric or no asymmetric volatility. If γ < 0, implies negative shock will increase the volatility more than positive shocks. If γ > 0, implies that positive shock increases the volatility more than negative shocks. According to previous studies in the subject, the coefficient γ is often negative, this implies that negative shocks have more impact on volatility than positive shocks (good news). Given that the model uses the log of the variance (σ^2), this means that even if the parameters are negative, the variance will still be positive. Therefore, the model is not subject to non-negative constraints. (B) Threshold GARCH Model (TGARCH).

The GJR-GARCH also called Treshold GARCH (TGARCH) model was developed by Glosten,

Jagannathan & Runkle (1993). The advantage of the model is that the variance is directly modelled and does not use the natural logarithm like the EGARCH model. The main target of the TGARCH model is to capture asymmetries in terms of negative and positive shocks. To do that is simply to add into the variance equation of the GARCH model a multiplicative dummy variable $\gamma\varepsilon_{t-i}I_{t-i}$ to check whether there is statistically significant difference when the shocks are negative. Hence the conditional variance for a TGARCH or GJR-GARCH is given by:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \, \sigma_{t-1}^2 + \sum_{i=1}^{m} \gamma$$

(3.9)

However, the form of GJR-GARCH (1, 1) is given by:

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-i}^2 + \beta\sigma_t^2 + \gamma\varepsilon_{t-i}I_{t-i}$$

(3.10)

In equation (3.9) above, σ_t^2 is the conditional forecasted variance, ω is the intercept for the variance, while $\alpha\varepsilon_{t-i}^2$ is the variance that depends on previous lag error term. β is the coefficient of previous period forecasted variance and σ_{t-1}^2 is the previous period forecasted variance. Moreso, γ is the scale (coefficient) of the asymmetric volatility. I_{t-i} is a dummy variable that is only activated if the previous shock is negative (ε_{t-i}< 0), allowing the GJR-GARCH to take the leverage effect into consideration. Glosten et al (1993).

$$I_{t-i} = \begin{cases} 1 \; if \; \varepsilon_{t-i} < 0 \text{ (Negative shock)} \\ 0 \; otherwise \; (Positive \; shock) \end{cases}$$

From equation (3.10), good news, (positive shock) and bad news, that is, negative shock.

(ε_{t-i}< 0) have different impacts on the conditional variance. A positive shock is captured by the coefficient α, ie have an α -effect on the conditional variance σ_t^2, while negative shock (Bad news) has an $(\alpha + \gamma)$ effect on the conditional variance σ_t^2, (volatility). If γ = 0, the GJR-GARCH model becomes a linear symmetric GARCH model but if γ

so: $\gamma > 0$, means negative shocks will increase $\neq 0$, then that suggests an asymmetric (leverage) effect. However, if al volatility more than positive shock.

If γ < 0, positive shock will increase volatility more than

negative shocks.

(C) The Power GARCH (PGARCH) Model

Ding et al (1993), expressed conditional variance using PGARCH (p,d,q) as;

$$\sigma_t^d = \beta_0 + \sum_{i=1}^{p} \alpha_i(|\varepsilon_{t-1}| + \gamma\varepsilon_{t-1})^d + \sum_{j=1}^{q} \beta_j\sigma_{t-j}^d$$

(3.11)

Here, d > 0 and $\gamma_i < 1$ establishes the existence of leverage effects. If d is set at 2, the PGARCH (p, q) replicates a GARCH(p , q) with a leverage effect. If d is set at 1, the standard deviation is modelled. The first order of the above PGARCH equation is PGARCH (1, d, 1) expressed as:

$$\sigma_t^d = \beta_0 + \alpha_1(|\varepsilon_{t-1}| + \gamma\varepsilon_{t-1})^d + \beta_1\sigma_{t-1}^d$$

(3.12)

The failure to accept the null hypothesis that $\gamma_1 \neq 0$ shows the presence of leverage effect. The impact of news on volatility in PGARCH is similar to that of TGARCH when d is 1.

7. Volatility Transmission (Spillover Effect)

The transmission of shocks from one market or variable to another was well documented by (Ewing, 2002). Co-movement across volatilities (co-volatility) due to common information that simultaneously affect expectations and information spillover caused by cross-market hedging are some of the reasons for volatility transmissions. In addition to endogenous events or variables, exogenous variables, deterministic events (macroeconomic announcements) may all have influence on the volatility process. To determine volatility transmission (Spillover) between variables (markets), we use the Augmented GARCH model as developed by (Duan, 1997). The model is defined as follows:

$$\sigma_t^2 = \alpha_0 + \beta\sigma_{t-1}^2 + \alpha\varepsilon_{t-1}^2 + \theta X_t \quad (3.13)$$

Where X_t is the residual squared error of ARMA model and θ the term that measures the magnitude of volatility spillover (transmission) across the variables (markets). Two variables, namely, exchange and inflation rates were introduced to determine whether they have any spillover effect on the conditional volatility of interest rate in Nigeria within the period under review.

8. Model Selection Criteria

Finding optimal of a model that will fit a particular data set has always been favourite for researchers. Reinhard and Lunde (2001), inferred that there is not a unique criterion for selecting the best model, rather it depends on preferences, example, expressed in terms of a utility function or loss function. The standard model selection criteria of (Akaike, 1974) are often applied. Bieren, H, J (2006) recommended the following modification, if the model includes ARCH type errors.

$$AIC = -2\log(\hat{\sigma}^2) + 2k - 1 - \log(2\pi) \quad (3.14)$$

Shibata (1976) showed through empirical evidence that AIC has the tendency to choose models which are over parameterized. Various modifications have been produced to overcome this lack of consistency. (Schwarz, 1978) developed a consistent criterion for models defined in terms of their posterior probability (Bayesian approach) which is given by;

$$SIC = -2log(\hat{\sigma}^2) + klog(n) \quad (3.15)$$

Where $(\widehat{\sigma^2})$ is the estimated model error variance, k is the number of free parameters in the model, n is the number of observations. In ARCH context, this form will look like;

$$SIC = -2log\left(\widehat{\sigma^2}\right) + klog(n) - 1 - \log(2\pi) \quad (3.16)$$

9. Loss Functions-Measure of Forecast Performance

Although in literature, several methods of measuring the performance conditional variance,

(Liu, 2009) inferred that the Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Mean Absolute Percent Error (MAPE) and Theil Inequality Coefficient (TIC) are most appropriate in determining the best forecast performance. (Clements, 2005), on the predictive ability of volatility models proposes that out-of-sample forecasting ability remains the criterion for selecting the best predictive model, hence shall be adopted in this study. If ${\sigma_t}^2$ and $\hat{\sigma}_t^2$ represents the actual and forecasted volatility of interest rate at time t, then;

$$RMSE = \sqrt{\frac{1}{T}\sum_{i=1}^{T}(\sigma_t^2 - \sigma_t^2)} \quad (3.17)$$

$$MAE = \frac{1}{T}\sum_{l=1}^{T} |\hat{\sigma}_t^2 - \sigma_t^2|$$

(3.18)

$$MAPE = \frac{1}{T}\frac{\sum_{i=1}^{T}(\hat{\sigma}_t^2 - \sigma_t^2)}{Z_t}$$

(3.19)

$$TIC = \frac{\sqrt{\frac{1}{T}\sum_{i=1}^{T}(\hat{\sigma}_t^2 - \sigma_t^2)^2}}{\sqrt{\frac{1}{T}\sum_{i=1}^{T}(\hat{\sigma}_t^2)^2} - \sqrt{\frac{1}{T}\sum_{i=1}^{T}(\sigma_t^2)^2}}$$

(3.20)

The volatility model with the least RMSE, MAE, MAPE and TIC statistic is the best forecasting model.

10. Distribution of Errors $\boldsymbol{\varepsilon}_t$

As far as error distribution is concerned, GARCH model theory suggests three assumptions about the distribution of residuals. These three assumptions may follow normal law, a student law or a generalized Error distribution (GED). In this work three different distributions namely; the normal (Gaussian), the student's t-Distribution and the Generalize Error distribution were employed to determine the best fit and forecast of the conditional variance. The Normal distribution:

$Z2$

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{}{2}}, \qquad -\infty < Z < \infty$$

(3.21)

$$f(Z) = \frac{\Gamma(\frac{v+1}{2})}{\sqrt{v\pi\Gamma(\frac{v}{2}}} (1 + \frac{Z^2}{v})^{-\frac{(v}{2}}$$

Student t-distribution:

+1)

$-\infty < Z < \infty$ (3.22)

Generalized Error Distribution:

$$\sigma^{-1} v e^{(-0>5)} |\frac{(\frac{z-\mu}{\sigma})}{\lambda}|$$

$$f(Z, \mu, \sigma, v) = \frac{\bar{v}}{\lambda \ ^{(1+\frac{1}{v})} \ (^1)} \quad 2 \quad \Gamma \frac{}{v}$$

(3.23)

V > 0 is the degree of freedom or the tail-tickness.

11. Mean Reversion

(John et al., 2019), stated that mean reversion means that current information has no influence on the long run forecast of the volatility. Persistence dynamics in volatility is generally captured in the GARCH coefficients of a stationary GARCH-type model. In stationary GARCH-type models, the volatility mean reverts to its long-run level, at a rate given by sum of ARCH and GARCH coefficients, which is usually close to one (1) for financial time series. The average number of time periods for the volatility to revert to its long run level is measured by the half-life of the volatility shock. The mean reverting form of the GARCH(1,1) model is given by;

$$\varepsilon_t^2 - \bar{\sigma}^2 = (\alpha + \beta)(\varepsilon_t^2{}_{-1} - \bar{\sigma}^2) + r_1 + \beta_1 + r_{1-1} \qquad (3.24)$$

Where $\bar{\sigma}^2 = \frac{\alpha_0}{1-\alpha-\beta}$, is the unconditional long-run level of volatility and $r_1 = (\varepsilon_t^2 - \bar{\sigma}^2)$. The magnitude of the mean reverting rate $\alpha + \beta$ (speed of adjustment) controls the speed of the mean reversion.

CONCLUSIONS

The target variable (interest rate) is the bank's lending rate. A total of forty-two models were estimated, with twenty-four symmetric and eighteen asymmetric models. The GED-GARCH (1,1) model emerged as the best fit, predicting that unconditional variance (homoscedasticity) would be achieved in the third month of the following year (March 2019). However, two independent variables, exchange and inflation rates, that were incorporated as external factors, were discovered to have an influence on the conditional variance of interest rates in Nigeria within the period under review. The interest rate in Nigeria is indeed volatile and the rate of decay of the shocks is very slow. The volatility is persistent and, as such, the best GARCH family model to adopt in analyzing the volatility of interest rates in Nigeria remains the symmetric GARCH (1,1) while the best error distribution is the generalized error distribution (GED).

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